

# (رياضيات الأعمال)

لناخصا رياضيات الأعمال مادة المحيد (شاطر 1 وشاطر 2)

\* يحتوي المناهج على شرح الدكتور / تيسير نور  
ويحتوي على التيسير بانك ويحتوي على شرح المادة وكل الجديد من أسئلة الامتحان

# بيكساز # واستمر كم فيها # لتكمل الصورة

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# انضموا لقروب المادة :- رياضيات الأعمال - بيكساز

رج تلاقوا كل شيء انجمن المادة موجود بالقروب

# خطية معادلات Linear Equations

CH-1

## Linear Equations :-

$$y = ax + b$$

معامل x      ثابت

# سطر  
# واسطه

ex:-  $y = 5x - 10$

$a = 5, b = -10$

X-intercept :  $y = 0$  متى يقطع الإحداثي لـ x المحور السيني  $(x, 0)$

Y-intercept :  $x = 0$  متى يقطع الإحداثي لـ y المحور الصادي  $(0, y)$

ex:- find X-intercept , Y-intercept for the equation  $y = 1 - 2x$

كذلك X-intercept  $\rightarrow y = 0$

$$y = 1 - 2x$$

$$0 = 1 - 2x$$

$$\frac{-1}{-2} = \frac{-2x}{-2}$$

$$x = \frac{1}{2}$$

Y-intercept  $\rightarrow x = 0$

$$y = 1 - 2x$$

$$y = 1 - 2(0)$$

$$y = 1$$

\* further Algebra اعزب من ايجر

1) Fractions كسور

$$1) \frac{2}{4} + \frac{1}{4} = \frac{3}{4}$$

$$2) \frac{4 \times 1}{4 \times 3} + \frac{1}{12} = \frac{4}{12} + \frac{1}{12} = \frac{5}{12}$$

الخ حالة الجمع والطرح يجب توصيل المقامات

$$3) \frac{2}{7} \times \frac{3}{4} = \frac{6}{28}$$

$$4) \frac{2}{7} \div \frac{3}{4} = \frac{2}{7} \times \frac{4}{3} = \frac{8}{21}$$

الخ حالة القسمة اكد الاول كما هو والقسمة تقابل الى ضرب واكد الثاني بقلبه

$$\begin{aligned}
 \text{ex: 1) } & \frac{1}{(x-1)(x+1)} - \frac{1}{(x-1)} \\
 &= \frac{1}{(x-1)(x+1)} - \frac{1(x+1)}{(x-1)(x+1)} \quad \text{نحوه مقادیر *} \\
 &= \frac{1}{(x-1)(x+1)} - \frac{(x+1)}{(x-1)(x+1)} \\
 &= \frac{1 - (x+1)}{(x-1)(x+1)} = \frac{1 - x - 1}{(x-1)(x+1)} = \frac{-x}{(x-1)(x+1)}
 \end{aligned}$$

$$\begin{aligned}
 2) \quad & \frac{xy}{(x-3)(x+2)} \div \frac{y^2}{(x+2)(x-1)^2 x} \\
 &= \frac{xy}{(x-3)\cancel{(x+2)}} * \frac{\cancel{(x+2)}(x-1)^2 x}{y^2 y} = \frac{x^2 * (x-1)^2}{(x-3)y}
 \end{aligned}$$

Solve: جدید مسئله

$$\begin{aligned}
 1) \quad & 2x - 1 = 3 \\
 & \quad \quad \quad +1 \quad \quad +1 \\
 & \frac{2x}{2} = \frac{4}{2} \\
 & \boxed{x = 2}
 \end{aligned}$$

$$\begin{aligned}
 3) \quad & 2x + 3 = 4x - 1 \\
 & \quad \quad \quad -2x \quad \quad -2x \\
 & 3 = 2x - 1 \\
 & \quad \quad \quad +1 \quad \quad +1 \\
 & \frac{2x}{2} = \frac{4}{2} \\
 & \boxed{x = 2}
 \end{aligned}$$

$$\begin{aligned}
 5) \quad & \sqrt{\frac{(x-1)}{2}} = 2 \\
 & \text{دست} \left( \sqrt{\frac{(x-1)}{2}} \right)^2 = (2)^2 \\
 & \frac{x-1}{2} = 4
 \end{aligned}$$

$$\begin{aligned}
 2) \quad & \frac{3}{x-1} = 5 \\
 & 5(x-1) = 3 \\
 & 5x - 5 = 3 \\
 & \quad \quad \quad +5 \quad \quad +5 \\
 & 5x = 8 \\
 & \frac{5x}{5} = \frac{8}{5} \\
 & \boxed{x = \frac{8}{5}}
 \end{aligned}$$

$$\begin{aligned}
 4) \quad & \frac{2}{2x+3} = \frac{3}{x} \\
 & 2x = 3(2x+3) \\
 & 2x = 6x + 9 \\
 & \quad \quad \quad -6x \quad \quad -6x \\
 & -4x = 9 \\
 & \quad \quad \quad -4 \quad \quad -4 \quad \quad -4 \\
 & x = \frac{9}{4}
 \end{aligned}$$

$$\begin{aligned}
 & \text{ضرب بجای} \left\{ \begin{aligned} x-1 &= 8 \\ \boxed{x &= 9} \end{aligned} \right.
 \end{aligned}$$



(3)

Q) Solve the system of the linear equation  
 or find the intersection (نقطة تقاطع) Point for  
 or find the Value of  $x$  or  $y$

سolving the question  
 نفس المطلوب

→ حل نظام المعادلات الخطية

1) elimination Method طريقة الحذف

2) substitution Method طريقة التعويض

\* Elimination Method طريقة الحذف

ex:  $X + 2y = 5$  Solve linear equations

$$2x + y = 1$$

$$\text{كل} \quad X + 2y = 5 \quad \rightarrow \quad X + 2y = 5 \quad \text{نجا المعادلات}$$

$$-2(2x + y = 1) \quad \rightarrow \quad -4x - 2y = -2$$

$$-3x = 3$$

$$\left[ X = -1 \right] \leftarrow \text{نفسها بأي معادلة معطاة بالسؤال}$$

لايجاد قيمة  $y$

$$X + 2y = 5$$

$$-1 + 2y = 5$$

$$2y = 6$$

$$\left[ y = 3 \right]$$

The Point is  $(X, y)$

$$(-1, 3)$$

التركيز هو أساس النجاح  
 في الدراسة، في التجارة، في العمل  
 وفي العلاقات الإنسانية  
 كافة



# \* Substitution Method طريقة الاستعاضة

ex:-  $X + 2y = 5$  Solve linear equations

خطوة (1)  $2X + y = 1$

Step 1:- اختيار أحد المعادلتين وجعل  $x$  أو  $y$  موضوع القانون

$$X + 2y = 5$$

خطوة (2)  $X = 5 - 2y$

Step 2:- استعاضة المعادلة الثانية (أي ما استخدمناه)

$$2X + y = 1$$

$$2(5 - 2y) + y = 1$$

$$10 - 4y + y = 1$$

$$10 - 3y = 1$$

$$-10 \quad -10$$

$$-3y = -9$$

$$\boxed{y = 3}$$

عوض بأي معادلة (قيمة  $y$ )  
مطلوب بالسؤال

$$X + 2y = 5$$

$$X + 2(3) = 5$$

$$X + 6 = 5$$

$$\boxed{X = -1}$$

ex) Solve the system:-

$$1) 3x - 2y = 4 \rightarrow 3x - 2y = 4$$

$$-1(x - 2y = 2) \rightarrow -x + 2y = -2$$

$$2x = 2$$

$$\boxed{x = 1}$$

$$3x - 2y = 4$$

$$3(1) - 2y = 4$$

$$3 - 2y = 4$$

$$-3 \quad -3$$

$$-2y = 1$$

$$\boxed{y = -\frac{1}{2}}$$

$$2) 2x + 4y = 6 \rightarrow 2x + 4y = 6$$

$$-2(x + 2y = 3) \rightarrow -2x - 4y = -6$$

$$0 = 0$$

Infinity many solutions  
فإنك حلول كثيرة

$$3) 2x + 4y = 6 \rightarrow 2x + 4y = 6$$

$$-2(x + 2y = 2) \rightarrow -2x - 4y = -4$$

$$0 \neq 2$$

إن كانت مساوية  
لا يوجد حلول

(No solve)

(5)

c) Solve the System

$$X + 3y - z = 4 \dots\dots\dots (1)$$

$$2X + y + 2z = 10 \dots\dots\dots (2)$$

$$3X - y + z = 4 \dots\dots\dots (3)$$

Step 1: أخذ معادلتين وحذف منهن متغير وتصبح معادلة رابعة

$$-2(X + 3y - z = 4) \rightarrow -2X - 6y + 2z = -8$$

$$2X + y + 2z = 10 \rightarrow 2X + y + 2z = 10$$

$$\boxed{-5y + 4z = 2} \rightarrow (4)$$

Step 2: استخدم المعادلة (1) ما استخدمناها في خطوة

الأولى و استخدم معها معادلة الـ (1) أو (2)

بما يجب حذف إحداهما بخطوة (1)

$$3X - y + z = 4 \rightarrow 3X - y + z = 4$$

$$-3(X + 3y - z = 4) \rightarrow -3X - 9y + 3z = -12$$

$$\boxed{-10y + 4z = -8} \rightarrow (5)$$

Step 3: معادلة (4) مع معادلة (5)

$$-1(-5y + 4z = 2) \rightarrow 5y - 4z = -2$$

$$-10y + 4z = -8 \rightarrow -10y + 4z = -8$$

$$\begin{array}{r} -15y = -10 \\ -5y = -10 \end{array}$$

$$\boxed{y = 2} \leftarrow \text{نستخدم معادلة (4) أو معادلة (5)}$$

$$-5y + 4z = 2$$

$$-5(2) + 4z = 2$$

$$-10 + 4z = 2$$

$$4z = 12$$

$$\boxed{z = 3}$$

بعد ما حللنا معادلتين من المعادلات

نستخدمها بأحد المعادلات

(نستخدم بالسؤال)

$$X + 3y - z = 4$$

$$X + 3(2) - 3 = 4$$

$$\boxed{X = 1}$$

(6)

ex) Solve:-

$$4x + y + 3z = 8 \rightarrow \textcircled{1}$$

$$-2x + 5y + z = 4 \rightarrow \textcircled{2}$$

$$3x + 2y + 4z = 9 \rightarrow \textcircled{3}$$

سر النجاح يكمن في التأقلم مع التغيرات

Step 1:- from equation  $\textcircled{1}$  and  $\textcircled{3}$ 

$$-2(4x + y + 3z = 8) \rightarrow -8x - 2y - 6z = -16$$

$$3x + 2y + 4z = 9 \rightarrow 3x + 2y + 4z = 9$$

$$\underline{-5x - 2z = -7} \rightarrow \textcircled{4}$$

Step 2:- from equation  $\textcircled{2}$  and  $\textcircled{1}$ 

$$-5(4x + y + 3z = 8) \rightarrow -20x - 5y - 15z = -40$$

$$-2x + 5y + z = 4 \rightarrow -2x + 5y + z = 4$$

$$\underline{-22x - 14z = -36} \rightarrow \textcircled{5}$$

Step 3:- from equation  $\textcircled{4}$  and  $\textcircled{5}$ 

$$-7 \times (-5x - 2z = -7) \rightarrow -35x + 14z = +49$$

$$-22x - 14z = -36 \rightarrow -22x - 14z = -36$$

$$\underline{13x = 13}$$

$$\boxed{x = 1}$$

Step 4:- from equation  $\textcircled{5}$ 

$$-22x - 14z = -36$$

$$-22(1) - 14z = -36$$

$$\begin{array}{r} -22 - 14z = -36 \\ +22 \quad \quad +22 \end{array}$$

$$-14z = -14$$

$$\boxed{z = 1}$$

Step 5:- from equation  $\textcircled{1}$ 

$$4x + y + 3z = 8$$

$$4(1) + y + 3(1) = 8$$

$$y + 7 = 8$$

$$\boxed{y = 1}$$

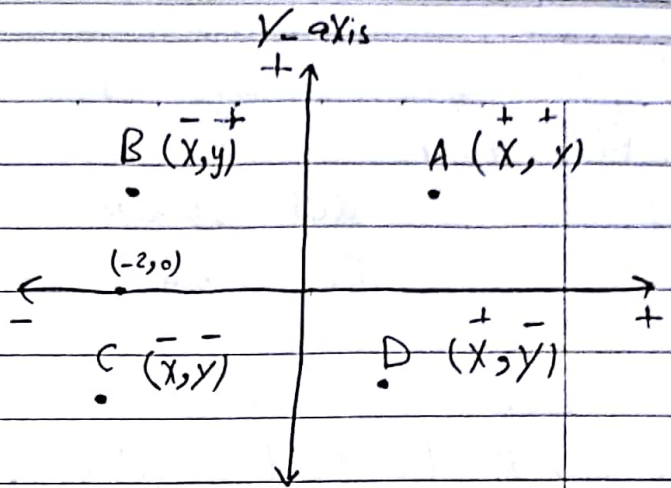


- Coordinate Plane: مسووی دیکارتی

أحداث سينية: بعد النكاح، الهجرات

أحداث صادقة: بعد النقطة عن كور السنين

- Coordinate systems  $\vec{r}, \vec{S}, \vec{g}$  same



Remark:-  $(x, y)$  is a coordinate of the point A where  $x$  is the  $x$ -coordinate and  $y$  is the  $y$ -coordinate

Q) Sketch the graph of the equation  $2x + 3y = 6$

دلیل  
step 1:- X-intercept:  $y=0 \Rightarrow 2x=6$

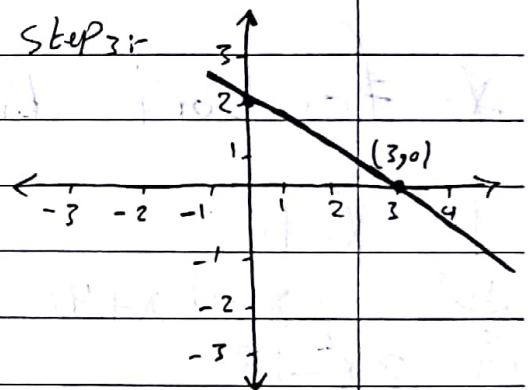
$$\Rightarrow \boxed{x=3}$$

$\Rightarrow$  Point is  $(3, 0)$

step 2: y-intercept:  $x=0 \Rightarrow 3y=6$

$$\Rightarrow \boxed{y=2}$$

$\Rightarrow$  Point is  $(0, 2)$



Q) sketch the graph of the equation  $2x + 3y = 6$

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step 1: جعل موضوع القانون

$$2x + 3y = 6$$

$$\frac{3Y}{3} = \frac{(6-2X)}{3} \Rightarrow Y = 2 - \frac{2}{3}X$$

$$a = \frac{-2}{3} < 0$$

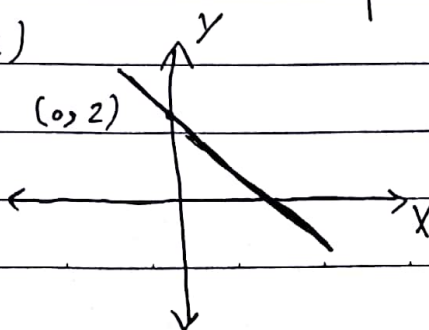
step 2:-  $y$ -intercept

$$y = 2 - \frac{2}{3} \quad (a)$$

$$y = 2$$

Point is  $(0, 2)$

steps:-



حسب الـ (مقابل اكس)

~~توزانہ~~ ~~Increase~~

اذا كان - 10/11 سنة

~~discrese~~  
~~veat~~

افقي horizontal

عمودي vertical

8

جد

احداثي

النقطة

Q) Find the coordinate of the one point on the line  $3x - 2y = 4$  when  $x = 4$

حل

$$3(4) - 2y = 4$$

$$12 - 2y = 4 \Rightarrow -2y = -8$$

$$y = 4$$

$$(4, 4)$$

نقطة

بقي

Q) Check the point  $(0, 0)$  و  $(2, 1)$  if lie on the line  $2x - y = 3$

حل

$$\begin{matrix} x & y \\ (0, 0) \end{matrix}$$

$$2x - y = 3$$

$$0 \neq 3$$

$$(2, 1)$$

$$2x - y = 3$$

$$2(2) - (1) = 3$$

$$3 = 3 \quad \checkmark$$

\* Factorization تقسيم العوامل

1)  $x^2 - 16$

حل  $(x - 4)(x + 4)$

2)  $2x^2 - 18$

حل  $2(x^2 - 9)$

$2(x - 3)(x + 3)$

3)  $x^2 + x - 12$

حل  $(x + 4)(x - 3)$



# \* Slope of a line (m)

- Slope of the line Passing through the Points A(x<sub>1</sub>, y<sub>1</sub>) and B(x<sub>2</sub>, y<sub>2</sub>)

is  $m = \frac{y_2 - y_1}{x_2 - x_1}$

Q) If the line Passing through the Points (-2, 3) and (5, 7)

Ans  $m = \frac{7-3}{5-(-2)} = \frac{4}{7}$

Slope of the line  $y = mx + c$  is m

Q) Find the slope of the line  $y = 3x - 1$

Ans  $m = 3$

Q) Find the slope of the line  $2x + 3y = 7$

Ans  $2x + 3y = 7$

$(3y = 7 - 2x) \div 3$

$y = \frac{7}{3} - \frac{2}{3}x$

$y = -\frac{2}{3}x + \frac{7}{3}$

Slope of this line is  $m = -\frac{2}{3}$

ليس من الخطأ  
أن تقول اراجع  
ما درست قد مشيت  
في طريق خاطئ

## \* equation and line

$y - y_1 = M(x - x_1)$

متطلبات معادلة الخط

1) نقطة (x<sub>1</sub>, y<sub>1</sub>)

2) M = ميل

Q) Find the equation of line which is

موازي لخط

الموازي

① Parallel to the  $3x + y = 1$  and Passing through the Point (0, 2)

is:

step 1:  $y - y_1 = M(x - x_1)$

$y - 2 = M(x - 0)$

$y - 2 = -3(x - 0)$

$y - 2 = -3x$

$y = -3x + 2$

step 3:  $M = -3$

Parallel  $M = M$

معادلة الخط الموازي

$3x + y = 1$   
 $y = -3x + 1$

$M = -3$

جواب  
معادلة الخط الموازي



بمس آشوف هاعطاه مباشره بحس  
معاداة خط

10

Q) Find the line of equation which is

المعاداة

② Perpendicular to the line  $3x+y=1$  and Passing through the Point  $(0,2)$  is.

دست

step 1:-  $y - y_1 = m(x - x_1)$

$y - 2 = m(x - 0)$

$y - 2 = \frac{1}{3}(x - 0)$

step 2:- Perpendicular

$y - 2 = \frac{1}{3}x$

step 3

معاداة الخط  $M \neq M = -1$  بالاسوال

$y = \frac{1}{3}x + 2$

دست

$3x + y = 1$

$y = 1 - 3x$

$y = -3x + 1$

$m = -3$

$M \neq M = -1$  معاداة الخط بالاسوال

$-3 \neq M = -1$  ماله

$m = \frac{1}{3}$

Q) Find The equation of line

المعاداة

③ intersection

Passing  $(2,3)$  and intersection of the line  $2x+3y=5$ ,  $3x-2y=1$

step 1:-  $y - y_1 = m(x - x_1)$

$\begin{cases} (x_1, y_1) \\ (2, 3) \\ (x_2, y_2) \\ (1, 1) \end{cases}$

نقطة التقاطع

step 2:-  $-3(2x+3y=5) \rightarrow -6x-9y=-15$

$2(3x-2y=1) \rightarrow 6x-4y=2$

$-13y=-13$

$(1,1)$

$y=1$

$3x-2(1)=1$

$3x-2=1 \rightarrow 3x=3 \rightarrow x=1$

step 3:  $M = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 3}{1 - 2} = \frac{-2}{-1} = 2$

$M = 2$

step 4:  $y - 3 = 2(x - 2)$

$y - 3 = 2x - 4$   
 $y = 2x - 1$

equation of line

$L \rightarrow \text{line}$   
 $L_1 \parallel L_2 \rightarrow M_1 = M_2$

$L_1 \perp L_2 \rightarrow M_1 = -\frac{1}{M_2}$

Q) IF  $L_1$  Passing through  $(0,0)$  and  $(1,1)$  find the following

1) equation of  $L_1$

step 1:  $y - y_1 = M(x - x_1)$

$\begin{matrix} \text{نقطتي} \\ \text{على الخط} \end{matrix} \begin{cases} (x_1, y_1) \\ (x_2, y_2) \end{cases} \begin{cases} (0,0) \\ (1,1) \end{cases}$

step 2:  $M = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 0}{1 - 0} = 1$

$M = 1$

step 3:  $y - 0 = 1(x - 0)$   
 $y = x$

2) equation of  $L_2 \perp L_1$  at  $(1,1)$

step 1:  $y - y_1 = M(x - x_1)$

step 2:

$M \times M = -1$   
 $1 \times M = -1$   
 $M = -1$

$M = -1$

step 3:  $y - 1 = -1(x - 1) \Rightarrow y - 1 = -x + 1 \Rightarrow y = -x + 2$

Ex 12  
 ③ Find the equation of  $L_2 \parallel L_1$  and Passing through  $(1, 4)$

step 1:  $Y - Y_1 = M(X - X_1)$

step 2:  $\frac{Y - Y_1}{\text{y-axis}} = \frac{M}{\text{slope}} = 1$

step 3:  $Y - 4 = 1(X - 1)$

$$\begin{array}{r} Y - 4 \\ +4 \end{array} = \begin{array}{r} X - 1 \\ +1 \end{array}$$

$$Y = X + 3$$

Q) Find an equation of line  $L$  with slope  $M = -\frac{1}{2}$  and Passing through the Point  $(-2, -3)$ .

step 1:  $Y - Y_1 = M(X - X_1)$

step 2:  $Y - (-3) = -\frac{1}{2}(X - (-2))$

$$Y + 3 = -\frac{1}{2}(X + 2)$$

$$\begin{array}{r} Y + 3 \\ -3 \end{array} = \begin{array}{r} -\frac{1}{2}X - 1 \\ +1 \end{array}$$

$$Y = -\frac{1}{2}X - 4$$

Ag

قمة الابداع  
 أن يخرج الإنسان  
 من وحل الفئس الى  
 إنسان يفرح في العمل



(13)

## \* Transposition of formula نقل الصيغة

Q1) write the equation  $2y = 5x - 10$  interms of  $x$ .

حل

$$\begin{array}{cc} 2y = 5x - 10 \\ +10 \quad +10 \end{array}$$

$$(2y + 10 = 5x) \div 5$$

$$\frac{2}{5}y + 2 = x$$

$$x = \frac{2}{5}y + 2$$

2) write interms of  $x$

$$y = \sqrt{\frac{x}{5}}$$

حل

$$y^2 = \frac{x}{5} \Rightarrow x = 5y^2$$

$$3) y = \frac{4}{3x+1}$$

حل

$$3xy + y = 4$$

$$\begin{array}{cc} -y & -y \end{array}$$

$$\frac{3xy}{3y} = \frac{4-y}{3y}$$

$$x = \frac{4}{3y} - \frac{1}{3}$$

$$4) y = \frac{2x+1}{3x-5}$$

حل

$$3xy - 5y = 2x + 1$$

$$-2x + 3xy = 5y + 1$$

$$\frac{x(-2+3y)}{-2+3y} = \frac{5y+1}{-2+3y}$$

$$x = \frac{5y+1}{3y-2}$$

Q) Make  $x$  the subject of formula:

$$1) y = \frac{1}{x(x-1)}$$

$$\underline{\text{Sol}} \quad \frac{1}{x}xy - y = 1$$

$$\frac{1}{x}xy = (1+y)$$

$$\frac{1}{x}y = \frac{(1+y)}{y}$$

$$x = \frac{1}{\frac{1}{y}} + \frac{1}{\frac{1}{y}}$$

$$3) y = \frac{\sqrt{x+1}}{2}$$

$$\underline{\text{Sol}} \quad (2y)^2 = (\sqrt{x+1})^2$$

$$4y^2 = x+1$$

$$x = 4y^2 - 1$$

*Shu*

$$2) y = \sqrt{\frac{x-1}{2}}$$

$$\underline{\text{Sol}} \quad y^2 = \left(\sqrt{\frac{x-1}{2}}\right)^2$$

$$y^2 = \frac{x-1}{2}$$

$$2y^2 = x-1$$

$$x = 2y^2 + 1$$

$$4) y = \frac{x+2}{x-2}$$

$$xy - 2y = x+2$$

$$xy - x = 2y+2$$

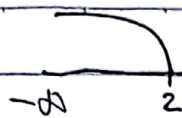
$$\frac{x(y-1)}{y-1} = \frac{(2y+2)}{y-1}$$

$$x = \frac{2y+2}{y-1}$$

# Inequalities

$>, <, \geq, \leq$

1)  $x < 2$



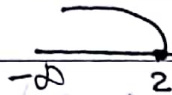
$(-\infty, 2)$

مفتوحة

بالمثل

عند عدم وجود مساواة تكون الفترة

2)  $x \leq 2$



$(-\infty, 2]$

عند وجود إشارة مساواة تكون الفترة مغلقة

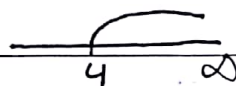
عند  $(\infty$  أو  $-\infty$ ) الفترة عندما مفتوحة (من جهة)

3)  $0 \leq x \leq 5$



$[0, 5]$

4)  $x > 4$



$(4, \infty)$

فترة مغلقة  $[a, b]$

$a \leq x \leq b$

$a \leq x < b$

فترة مفتوحة  $(a, b)$

$a < x < b$

$a < x \leq b$

\* تغيير إشارة المتباينة عند الضرب بعدد سالب

$7 > 3 \xrightarrow{\text{ضرب سالب واحد}} -7 < -3$

\* عند قلب العدد تغيير إشارة المتباينة

$22 > 2 \xrightarrow{\text{قلب الأعداد}} \frac{1}{22} < \frac{1}{2}$

\* تغيير إشارة المتباينة عند القسمة على عدد سالب

$10 > 2 \xrightarrow{\text{تقسيم -2}} -5 < -1$

\* عند جمع أو طرح لا تتأثر إشارة المتباينة

$7 > 2 \xrightarrow{\text{إضافة 2}} 9 > 4$

$7 > 2 \xrightarrow{\text{طرح 2}} 5 > 0$

\* عند ضرب أو قسمة (أعداد موجبة) فإن إشارة المتباينة لا تتغير

$7 > 2 \xrightarrow{\text{ضرب 2}} 14 > 4$

$25 > 5 \xrightarrow{\text{قسمة 5}} 5 > 1$



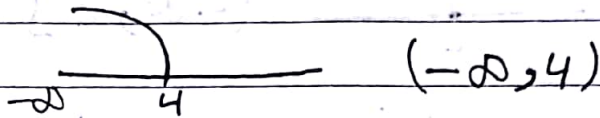
(16)

$$Q) \frac{-4x+5}{-5} > \frac{-11}{-5}$$

JST

$$\frac{-4x}{-4} > \frac{-16}{-4}$$

$$x < 4$$



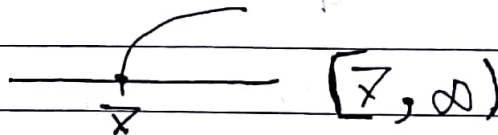
$$Q) 3x+5 \leq 4x-2$$

JST

$$\frac{3x}{-4x} \leq \frac{4x-7}{-4x}$$

$$-x \leq -7$$

$$x \geq 7$$



$$Q) \frac{-4}{-1} \leq \frac{3x+1}{-1} \leq \frac{7}{-1}$$

JST

$$\frac{-5}{3} \leq \frac{3x}{3} \leq \frac{6}{3}$$

$$\frac{-5}{3} \leq x \leq 2$$

$$\left[ \frac{-5}{3}, 2 \right]$$

# \* Absolute Value Function $y = |x|$ اقتران القيمة المطلقة $y = |x|$

$$|x| = \begin{cases} x & : x \geq 0 \\ -x & : x < 0 \end{cases}$$

$$|3| = 3$$

$$|-3| = 3$$

$$|x| = |-x|$$

Rule (1):  $|x| = a \Rightarrow x = a \text{ or } x = -a$

ex)  $|x| = 2 \Rightarrow x = 2 \text{ or } x = -2$

Q<sub>1</sub>) Solve  $|x-3| = 5$

sol  $x-3 = 5 \text{ or } x-3 = -5$   
 $x = 8 \text{ or } x = -2$

Solution is  $\{-2, 8\}$

Q<sub>2</sub>) Solve  $|3x+2|-5 = 7$

sol  $|3x+2| = 12$

$3x+2 = 12 \text{ or } 3x+2 = -12$

$3x = 10 \text{ or } 3x = -14$

$x = \frac{10}{3} \text{ or } x = \frac{-14}{3}$

Solution is  $\left\{\frac{-14}{3}, \frac{10}{3}\right\}$

Q<sub>3</sub>) Solve  $|x-3| \leq 5$

Rule (2):  $|f(x)| \leq b = -b \leq f(x) \leq b$

sol  $-5 \leq x-3 \leq 5$   
 $+3 \quad +3 \quad +3$

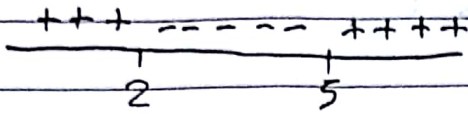
$-2 \leq x \leq 8$

Solution is  $[-2, 8]$

Rule (3):  $|f(x)| \geq b \Rightarrow f(x) \geq b \text{ or } f(x) \leq -b$

Q) Solve  $(x-2)(x-5) \geq 0$

حل



# واستقر في

$$x=1 \Rightarrow (-)(-) \Rightarrow +$$

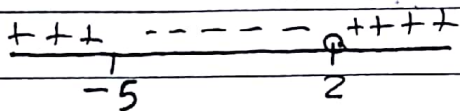
$$x=3 \Rightarrow (+)(-) \Rightarrow -$$

$$x=6 \Rightarrow (+)(+) \Rightarrow +$$

Solution is  $(-\infty, 2] \cup [5, \infty)$

Q) Solve  $\frac{x+5}{x-2} \leq 0$

حل



مع السؤال توجد إشارة المساواة

في المتباينة لكن لا تكون مغلقة

من جهة ال (2) لأنه صفر مقام

$$x=-6 \Rightarrow \frac{(-)}{(-)} = (+)$$

$$x=0 \Rightarrow \frac{(+)}{(-)} = (-)$$

$$x=3 \Rightarrow \frac{(+)}{(+)} = +$$

Solution is  $[-5, 2)$

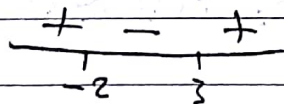
Q) Solve  $x^2 - x - 6 > 0$  (حدد قيم x حيث يكون 6 والفقر بينهم 1)

حل

$$(x+2)(x-3) > 0$$

ننظر إلى إشارة ال x اذا

كانت سالبة أو موجبة تكون إشارة العدد الأكبر



$$x=-3 \Rightarrow (-)(-) = (+)$$

$$x=2 \Rightarrow (+)(-) = (-)$$

$$x=4 \Rightarrow (+)(+) = (+)$$

Solution is  $(-\infty, -2) \cup (3, \infty)$



# \* Supply and Demand functions

A function  $f: X \rightarrow Y$  is a relation between two set  $X$  and  $Y$  such that every element in  $X$  has one and only one image in  $Y$ .

$\Rightarrow y = f(x)$  is a function.

ex)  $y = f(x) = x^2$   
 $f(2) = 2^2 = 4$

Quantity  $\text{الكمية}$   
 $Q$

\* supply and demand functions:

$P = aQ_D + b \rightarrow$  Demand function

$P = aQ_S + b \rightarrow$  supply function

decrease function

$a < 0 \rightarrow$  Demand function

$a > 0 \rightarrow$  supply function

Increase function

$P = aQ + b$   
 $\downarrow$  Price  $\downarrow$  Quantity

Q) The demand and supply function are

$P = -2Q + 50$  demand function  
 $P = \frac{1}{2}Q + 25$  supply function

Find: 1) The equilibrium Point

At  $Q_D = Q_S = Q$

At equilibrium Point  $Q_D = Q_S = Q$

نقطة التوازن  $P$  هي النقطة التي يتساوى فيها العرض والطلب  
 إذاً  $Q_D = Q_S = Q$

$Q_D = Q_S$

$-2Q + 50 = \frac{1}{2}Q + 25$

$-2Q = \frac{1}{2}Q - 25$

$-\frac{1}{2}Q = -\frac{25}{2}$

$-\frac{1}{2}Q - 2Q = -25$

$-\frac{5}{2}Q = -25$

$Q = 10$

نقطة التوازن

نقطة التوازن

equilibrium Point  $(Q, P)$

النقطة  $(Q, P)$

$-\frac{1}{2} - \frac{2 \times 2}{1 \times 2}$   
 $-\frac{1}{2} - \frac{4}{2} = -\frac{5}{2}$

$P$   
 السعر  
 $P = \frac{1}{2}Q + 25$   
 $P = \frac{1}{2}(10) + 25$   
 $5 + 25 = 30$  is the Price at equilibrium  
 $\Rightarrow$  At equilibrium  $(Q, P) = (10, 30)$

ضريبة

2 Find the new equilibrium Point tax 5 Jd for each good

كل  $P = -2Q_d + 50$

من  $P$  موفوق (قانونه)

$P = \frac{1}{2}Q_s + 25$

الضريبة تضاف على اقتران العرض (تضاف للإقتران بعد المساواة)

$P = \frac{1}{2}Q_s + 25 + 5$

$P = \frac{1}{2}Q_s + 30$

$P = -2Q_d + 50$

$P = \frac{1}{2}Q_s + 30$

$Q_s = Q_d = Q$

$-2Q + 50 = \frac{1}{2}Q + 30$

$-2Q + 20 = \frac{1}{2}Q$

$-\frac{2}{5}(-5Q) = (-20)(-\frac{2}{5})$

$Q = 8$

$\Rightarrow P = \frac{1}{2}Q_s + 30$

$= \frac{1}{2}(8) + 30 = 34$

$\Rightarrow$  At equilibrium  $(Q, P) = (8, 34)$

جدال  $P$  كوفيا كالمعادنة

(بإقترانه supply جديد)

أو demand (قديم)

equilibrium Price  
 هوون السعر المتوازن  
 equilibrium Quantity  
 هوون كمية المتوازن



ملاحظة: نقطة التوازن بعد الضريبة (8, 34)

نقطة التوازن قبل الضريبة (10, 30)

يمكننا إيجاد الفرق في السعر بين قبل الضريبة وبعدها الضريبة

$$P = 4 \text{ Jd}$$

الذي يتحمل فرق السعر هو Consumer

ex)  $P = -3Q + 51$

1) find Q when  $P = 0$

$$0 = -3Q + 51$$

$$-51 = -3Q$$

$$Q = 17$$

2) find P when  $Q = 1$

$$P = -3(1) + 51$$

$$= 48$$

Q) Multiply  $(2x-3)(x^2+2x+5)$

$$\text{كس} \quad 2x(x^2+2x+5) - 3(x^2+2x+5)$$

$$= 2x^3 + 4x^2 + 10x - 3x^2 - 6x - 15$$

$$= 2x^3 + x^2 + 4x - 15$$

Q) find  $(x+3)^2$

$$\text{كس} \quad x^2 + 6x + 9$$

Q)  $(3x-2)^2$

$$\text{كس} \quad 9x^2 - 12x + 4$$

$$(x+y)^2 = x^2 + 2xy + y^2$$

$$(x-y)^2 = x^2 - 2xy + y^2$$

Q) without using a calculator the exact value of  $(2\frac{1}{4})^{-\frac{1}{2}}$

$$\text{كس} \quad \left(2\frac{1}{4}\right)^{-\frac{1}{2}} = \left(\frac{9}{4}\right)^{-\frac{1}{2}} = \left(\frac{4}{9}\right)^{\frac{1}{2}} = \frac{\sqrt{4}}{\sqrt{9}} = \frac{2}{3}$$

Q) IF  $x = -3$ ,  $y = 4$  and  $z = -2$  evaluate.

$$3x^2 + 9y - 6z = 3(-3)^2 + 9(4) - 6(-2)$$

$$= 27 + 36 + 12 = 75$$



Q) If the equation  $2x+4 = 4(x-2)$   
and  $-x+k = 2x+1$  have the same solutions  
then the value of  $k =$

$$\begin{aligned} \text{Sol} \quad 2x+4 &= 4(x-2) \\ 2x+4 &= 4x-8 \\ 2x &= 4x-12 \\ -4x & \quad -4x \\ -2x &= -12 \\ x &= 6 \end{aligned}$$

$$-x+k = 2x+1$$

$$-6+k = 12+1$$

$$k = 19$$

Q)  $\left| \frac{x}{2} - 3 \right| \leq 4$

$$\text{Sol} \quad -4 \leq \frac{x}{2} - 3 \leq 4$$

$$-1 \leq \frac{x}{2} \leq 7$$

$$-2 \leq x \leq 14$$

$$[-2, 14]$$

Q)  $3 + |x+4| = 5$

$$\text{Sol} \quad |x+4| = 2$$

$$x+4 = 2$$

or

$$x+4 = -2$$

$$x = -2$$

or

$$x = -6$$

Solution is  $\{-2, -6\}$

Q) Find the slope of the line  $10x - 3y - 7 = 0$

$$\text{Sol} \quad -3y = -10x + 7$$

$$y = \frac{10}{3}x - \frac{7}{3}$$

$$m = \frac{10}{3}$$

## أسئلة متنوعة عن شابر 1

Q1)  $f(x, y) = x^2 + 2y$  ,  $f(1, 3)$

ج1  $= 1^2 + 2(3) = 1 + 6 = 7$

Q2)  $f(x, y, z) = 2x + y - z$  ,  $f(0, -2, -1)$

$= 2(0) + (-2) - (-1) = -2 + 1 = -1$

Q3)  $y = \sqrt{\frac{x-1}{x+2}}$  , Make  $x$  of subject.

ج1  $y^2 = \frac{x-1}{x+2}$

$xy^2 + 2y^2 = x - 1$

$xy^2 - x = -2y^2 - 1$

$x(y^2 - 1) = -2y^2 - 1$

$x = \frac{-2y^2 - 1}{y^2 - 1}$

Q4)  $y = 3x^2 + 5$  , Make  $x$  of subject

ج1  $\frac{y-5}{3} = x^2 \Rightarrow \frac{y-5}{3} = x^2$

$\sqrt{\frac{y-5}{3}} = \sqrt{x^2}$

$x = \sqrt{\frac{y-5}{3}}$

Q5) The Product of  $(x-2)^2(x-1)$

ج1  $(x^2 - 4x + 4)(x-1)$

$= x^3 - 4x^2 + 4x - x^2 + 4x - 4$

$= x^3 - 5x^2 + 8x - 4$

Q6) Express  $X$  in terms of  $y$  the formula  $y = 2 - \sqrt{2x-3}$   
 $X =$

Sol  $y = 2 - \sqrt{2x-3}$

$$(y-2) = -\sqrt{2x-3} \quad * -1$$

$$(-y+2)^2 = (\sqrt{2x-3})^2$$

$$(-y+2)^2 = 2x-3$$

$$(-y+2)^2 + 3 = 2x$$

$$x = \frac{(-y+2)^2 + 3}{2}$$

is it right?

Q7) The equation of the line that has  $y$ -intercept 2 and Perpendicular to the line  $8x+2y=1$  is:

Sol  $8x+2y=1$

$$(2y=1-8x) \div 2$$

$$y = \frac{1}{2} - 4x$$

$$M \times M = -1$$

$$-4 \times M = -1$$

$$M = \frac{1}{4}$$

is it right?  $m = -4$

$$y-y_1 = M(x-x_1)$$

$$y-2 = \frac{1}{4}(x-0)$$

$$y-2 = \frac{1}{4}x$$

$$y = \frac{1}{4}x + 2$$

$y$ -intercept 2  
 $\downarrow$   $x=0$   $\downarrow$   $y=2$   
 $(0, 2)$

Q8)  $\frac{2}{x-1} = \frac{4}{x+4}$  then  $x =$

Sol  $14x+8 = 4x-4$

$$14x-4x = -8-4$$

$$10x = -12$$

$$x = -1.2$$



Q9) The solution of the equation  $\sqrt{\frac{3x}{x-6}} = 3$

$$\underline{\text{Sol}} \quad \frac{3x}{x-6} = \frac{9}{1}$$

$$3x = 9x - 54$$

$$-6x = -54$$

$$x = 9$$

Q10) without using a calculator, the exact value of  $(1\frac{11}{25})^{\frac{-1}{2}}$  is

$$\underline{\text{Sol}} \quad (1\frac{11}{25})^{\frac{-1}{2}} \Rightarrow (\frac{36}{25})^{\frac{-1}{2}} = (\frac{25}{36})^{\frac{1}{2}} = \sqrt{\frac{25}{36}}$$

$$\frac{\sqrt{25}}{\sqrt{36}} = \frac{5}{6}$$

Q11) If  $P = -6Q_d + 48$  and  $P = Q_s + 23$  the equilibrium quantity if the government imposes a fixed tax of 4 Jd of each good is:

$$\underline{\text{Sol}} \quad P = -6Q_d + 48$$

$$P = Q_s + 23 + 4$$

$$Q_s = Q_d = Q$$

$$-6Q + 48 = Q + 27$$

$$-7Q = -21 \quad Q = 3$$

Q12) If  $x=2$ ,  $y=3$  and  $z=4$  then the value of  $3x-2y+5z=$

$$\underline{\text{Sol}} \quad 3(2) - 2(3) + 5(4) = 6 - 6 + 20 = 20$$

Q13) If  $\frac{-3}{q-2} = \frac{3}{q+2}$  then  $q =$

$$\underline{\text{Sol}} \quad -3(q+2) = 3(q-2)$$

$$-3q - 6 = 3q - 6$$

$$-3q = 3q + 6 - 6$$

$$6q = 0$$

$$q = 0$$

Q14) IF  $X = \sqrt{2X+3}$  then X

$$\text{Sol } (X)^2 = (\sqrt{2X+3})^2$$

$$X^2 = 2X+3$$

$$X^2 - 2X - 3 = 0$$

$$(X-3)(X+1)$$

$$X=3, X=-1$$

Q15)  $|5X+4| > 9$

$$\text{Sol } 5X+4 > 9 \quad \text{or} \quad 5X+4 < -9$$

$$5X > 5 \quad \text{or} \quad 5X < -13$$

$$X > 1 \quad \text{or} \quad X < \frac{-13}{5}$$

Q16) The solution of  $\frac{9(\frac{X}{3})^2}{(\sqrt{3})^6} = 81$  is X =

$$\text{Sol } \frac{9 \cdot \frac{2 \cdot 2X}{3 \cdot 3}}{(\frac{1}{3})^6} = 81 \quad \Rightarrow \quad \frac{9 \cdot \frac{2 \cdot 2X}{3 \cdot 3}}{3} = 81$$

$$\frac{2+2X}{3} = 3$$

$$\Rightarrow \frac{2+2X}{3} = 3$$

$$2+2X = 7$$

$$2X = 5$$

$$X = \frac{5}{2}$$

Q1) IF the demand and supply functions of a good are  
 $P = -3Q_d + 54$  ,  $P = Q_s + 26$

a) The market equilibrium point  $(Q, P) =$

b) If a fixed tax of 4 Jd is imposed then the new equilibrium point  $(Q, P) =$

Q2) IF  $Y = 1 + \sqrt[4]{X+3}$  , then  $X =$  (Make X the subject)

Sol

$$(Y-1) = \sqrt[4]{X+3}$$

$$(Y-1)^4 = X+3$$

$$(Y-1)^4 - 3 = X$$

$$X = (Y-1)^4 - 3$$

Q3) Express X in terms of Y the formula  $Y = \frac{2-X}{X+1}$  ,  $X =$

Q4) Solve the equation  $\frac{8}{X-2} - \frac{13}{2} = \frac{3}{2X-4}$

Sol

$$\frac{8}{X-2} - \frac{3}{2X-4} = \frac{13}{2}$$

$$\left( \frac{8}{X-2} - \frac{3}{2(X-2)} = \frac{13}{2} \right) \times 2$$

$$\frac{16}{X-2} - \frac{2 \times 3}{2(X-2)} = \frac{13 \times 2}{2} \Rightarrow \frac{16}{X-2} - \frac{3}{X-2} = 13$$

$$\frac{16-3}{X-2} = 13 \Rightarrow \frac{13}{X-2} = 13$$

$$13X - 26 = 13 \Rightarrow 13X = 39$$

$$X = 3$$



Q5) The Product of  $(x-3)(x-2) =$

Q6) The solution of  $|x+3| = 11$  is  $x =$

Q7) Express  $x$  in terms of  $y$  the formula  $y = 2 - \sqrt{x-3}$ ,  $x =$

Q8) Solve the inequality  $(x-2)(x-4) \leq 0$

Q9) Find the equation of line has  $y$ -intercept  $= 5$  and Parallel to the line  $2x + 4y = 5$

Q10) If  $y = \frac{x+1}{x-1}$ , then  $x$  in terms of  $y$  is written as  $x =$

Q11) If the demand and supply function are  $= -3Q_d + 25$  and  $P = 2Q_s + 7$  find the equilibrium point after imposing a fixed tax  $= 3$   $\text{JD}$

Q12) The solution of  $|2x+3| \leq 5$  is

Q13) The Product of  $(x-2)^2(x-1)$

Q14) The solution of  $|2x-3| = 11$  is  $x =$

Q15) If the equation  $3x+6 = 2(2x+2)$  and  $-x+k = 2x-1$  have the same solutions, Then the value of  $k =$

Q16) The equation of line that has  $y$ -intercept 5 and Perpendicular to the line  $4x+2y=1$  is

Q17) If  $X = -3$ ,  $Y = 4$  and  $Z = -2$  evaluate  $2X^2 + 3Y - 5Z$

Q18) If the demand and supply functions of a good are given by

$$P = -3Q_d + 48, P = 1Q_s + 23$$

Then the equilibrium quantity<sup>2</sup> if the government imposes a fixed tax of 4 J.d on each good is =

Q19) express  $X$  in terms of  $Y$  the formula  $Y = \frac{2+X}{1-X}$

Q20) If  $X = \frac{-1}{2}$  and  $Y = \frac{1}{4}$  then  $3X - 6Y + XY =$

Q21) If  $Y = \frac{X-5}{3-2X}$ , then  $X$  in terms of  $Y$  is  $X =$

Q22) The solution of  $(X-4)(X+1) \leq 0$  is

Q23) The solution of  $\frac{2}{3-X} = \frac{5}{2X+4}$  is  $X =$

Q24) Point the intersection for  $y + 2x = 24$  and  $y - 4x = 12$

Q25) If  $Y = \frac{3+X}{3-X}$ ,  $X$  in terms of  $Y$  is  $X =$

Q26)  $C = 5$ ,  $D = 2$ ,  $g = 1$  evaluate  $2C - 3g + D$

Q27)  $|X+9| = 15$  find  $X$

Q28)  $2(X-3) = X+2$  find  $X$

Q29) Solve the inequality  $\frac{X+2}{3X-1} \geq 0$



Q30) Solve the inequality  $\frac{2x-1}{4} \geq \frac{x-2}{2}$

Q31) If  $6x - 3x + 3y = 2x$  find the slope

Q32) Express  $x$  in terms of  $y$   
 $y = 3 + \sqrt{x - \frac{1}{3}}$

Q33) The solution of  $\frac{2}{3-x} = \frac{5}{2x+4}$  is  $x =$

Q34) The Product  $3x(x-1)(x^2+2)$

Q35) If  $x=2, y=3$  then  $\frac{1}{xy} + \frac{1}{y} + \frac{1}{x} =$

Q36) The solution of  $|4-3x| < 5$

Q37) The solution of  $|5x+4| > 9$

Q38) find equation of  $L_2$  if  $L_2$  perpendicular to the line (1) that passing through  $(0,0), (1,1)$  at  $(1,1)$

Q39) Demand function  $P = 4Q + 100$

1) find  $P$  at  $Q=20$

2) find  $Q$  at  $P=200$

Q40) find The Coordinates of the Points where the line  $x-2y=2$

Q41) find the Point of intersection of the two lines

$$4x + 3y = 11$$

$$2x + y = 5$$



Q42) Solve the equation  $3X + 5Y = 19$   
 $-5X + 2Y = -11$

by eliminating one of the variables (مُزيل)

Q43) Solve the equation  
 $4X + Y + 3Z = 8$

$$-2X + 5Y + Z = 4$$

$$3X + 2Y + 4Z = 9$$

Q44) The demand and supply functions for two interdependent commodities are given by

$$Q_{D1} = 10 - 2P_1 + P_2$$

$$Q_{D2} = 5 + 2P_1 - 2P_2$$

$$Q_{S1} = -3 + 2P_1$$

$$Q_{S2} = -2 + 3P_2$$

Determine the equilibrium price and quantity for this two commodity model.

Sol The equilibrium  $Q_{D1} = Q_{S1}$  and  $Q_{D2} = Q_{S2}$

$$Q_1 = 10 - 2P_1 + P_2$$

$$Q_1 = -3 + 2P_1$$

$$\Rightarrow 10 - 2P_1 + P_2 = -3 + 2P_1$$

$$\boxed{-4P_1 + P_2 = -13}$$

$$Q_2 = 5 + 2P_1 - 2P_2$$

$$Q_2 = -2 + 3P_2$$

$$\Rightarrow 5 + 2P_1 - 2P_2 = -2 + 3P_2$$

$$\boxed{2P_1 - 5P_2 = -7}$$

$$-4P_1 + P_2 = -13 \rightarrow -4P_1 + P_2 = -13$$

$$2 \times (2P_1 - 5P_2 = -7) \rightarrow 4P_1 - 10P_2 = -14$$

$$-9P_2 = -27 \Rightarrow P_2 = 3$$

$$-4P_1 + 3 = -13 \Rightarrow -4P_1 = -16 \Rightarrow P_1 = 4$$

النتيجة:  $P_1 = 4$  و  $P_2 = 3$

32

$$P_1 = 4, P_2 = 3$$

$$QD_1 = 10 - 2(4) + 3 = 5 \checkmark$$

$$QD_2 = 5 + 2(4) - 2(3) = 7 \checkmark$$

$$QS_1 = -3 + 2(4) = 5 \checkmark$$

$$QS_2 = -2 + 3(3) = 7 \checkmark$$

# Ch-2 Quadratic functions

①

اقترانات تربيعية

Quadratic functions  
(Non-linear functions)

$$f(x) = ax^2 + bx + c$$

$$a \neq 0$$

# Quadratic equations

معادلات تربيعية

$$ax^2 + bx + c = 0$$

① Solve  $x^2 + x - 6 = 0$

تحليل بالعوامل

$$(x+3)(x-2) = 0$$

$$x+3=0 \text{ or } x-2=0$$

$$x = -3$$

$$x = 2$$

Solution is  $\{-3, 2\}$

قانون

\* General formula for solving  $ax^2 + bx + c = 0$  is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$D = b^2 - 4ac$$

① Solve  $x^2 + x - 6 = 0$

$$a = 1$$

$$b = 1$$

$$c = -6$$

$$D = 0 \rightarrow \text{one solution}$$

$$D > 0 \rightarrow \text{two solution}$$

$$D < 0 \rightarrow \text{No solution}$$

$$D = b^2 - 4ac$$

$$= (1)^2 - 4(1)(-6)$$

$$= 1 + 24 = 25 > 0$$

$$D > 0 \rightarrow \text{two solution}$$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$$2a$$

$$x = \frac{-1 + \sqrt{25}}{2}$$

$$x = \frac{-1 + 5}{2} = \frac{4}{2} = 2$$

$$x = \frac{-1 - \sqrt{25}}{2}$$

$$x = \frac{-1 - 5}{2} = \frac{-6}{2} = -3$$

Solution is  $\{-3, 2\}$

D:- Discriminant

التمييز



② Solve  $2x^2 - 5x - 12 = 0$

Sol

$$D = b^2 - 4ac$$

$$= (-5)^2 - 4(2)(-12)$$

$$= 25 + 96$$

$$= 121 > 0 \rightarrow \text{two solutions}$$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$$x = \frac{-b + \sqrt{D}}{2a}$$

$$= \frac{5 + 11}{4} = \frac{16}{4} = 4$$

$$x = \frac{-b - \sqrt{D}}{2a}$$

$$x = \frac{5 - 11}{4} = \frac{-6}{4} = \frac{-3}{2}$$

Solution is  $\left\{ \frac{-3}{2}, 4 \right\}$

②  $2x^2 - 5x - 12 = 0$

$$(2x+3)(x-4)$$

$$2x+3=0 \quad \left\{ \quad x-4=0 \right.$$

$$2x=-3 \quad \left\{ \quad x=4 \right.$$

$$x = \frac{-3}{2}$$

\* Discriminant  $D = b^2 - 4ac$  for solving  $ax^2 + bx + c = 0$

1) If  $D > 0$ , then the equation  $ax^2 + bx + c = 0$  has two solutions

2) If  $D = 0$ , then the equation has one solution

ex) solve this  $(x-2)^2 = 0$

$$\underline{\text{Sol}} \quad x^2 - 4x + 4 = 0 \Rightarrow x = 2$$

③ If  $D < 0$ , then the equation has no solution

ex) solve this  $x^2 + 4 = 0$

$x^2 = -4$  has no solution in real numbers

(3)

ex) The equation  $X^2 + CX + 2C = 0$  has one solution  
Find the value of C?

دست  $D = b^2 - 4ac$   $a = 1$   $D = 0$   $a \neq 0$   
 $0 = C^2 - 4(1)(2C)$   $b = C$   
 $0 = C^2 - 4(2C)$   $C = 2C$   
 $0 = C^2 - 8C$   
 $C^2 - 8C = 0$

$$C(C - 8) = 0$$

$$C = 0 \text{ or } C = 8$$

ex)  $(X^2 - 5X + 6) = 0$

دست  $(X - 2)(X - 3) = 0$

$$X - 2 = 0 \quad \left\{ \quad X - 3 = 0 \right.$$

$$X = 2 \quad \left\{ \quad X = 3 \right.$$

ex) If the equation  $X^2 - 4X + C = 4X - C$  has exactly one solution  
Then value of C = \_\_\_\_\_

دست  $X^2 - 4X + C = 4X - C$   
 $X^2 - 8X + 2C = 0$

$$a = 1$$

$$b = -8$$

$$C = 2C$$

$$D = b^2 - 4ac$$

$$0 = 64 - 4(1)(2C)$$

$$0 = 64 - 8C$$

$$8C = 64$$

$$C = 8$$

ex) If  $X + \frac{1}{X} = 6$  Then  $\left(X + \frac{1}{X}\right)^2$   $\leftarrow$   $\frac{1}{X^2}$

دست  $\left(X + \frac{1}{X}\right)^2 = (6)^2$

$$X^2 + 2\left(X\right)\left(\frac{1}{X}\right) + \frac{1}{X^2} = 36$$

$$X^2 + 2 + \frac{1}{X^2} = 36 \Rightarrow X^2 + \frac{1}{X^2} = 34$$

(4)

ex) If  $(X+y)^2 = 35$  and  $(X-y)^2 = 25$  then  $X^2 + y^2$

sol  $(X+y)^2 = 35$  ,  $(X-y)^2 = 25$

$$X^2 + 2XY + y^2 = 35$$

$$X^2 - 2XY + y^2 = 25$$

$$2X^2 + 2y^2 = 60$$

$$2(X^2 + y^2) = 60$$

$$(X^2 + y^2) = 30$$

ex) If one solution of the equation  $X^2 - 10X + C = 0$  is  $X = 2$  then the second solution is  $X = \underline{\hspace{2cm}}$

sol  $X^2 - 10X + C = 0$

$$(2)^2 - 10(2) + C = 0$$

$$\boxed{C = 16}$$

$$X^2 - 10X + 16 = 0$$

$$(X-8)(X-2)$$

$$\underline{X=8} \quad \underline{X=2}$$

حل السؤال  $\downarrow$  الجواب

ex) If equation  $X^2 + 2kX + 5k = 0$  has only one zero then  $k$ .

sol  $\Delta = D = b^2 - 4ac$

$$0 = (2k)^2 - 4(1)(5k)$$

$$0 = 4k^2 - 20k$$

$$4k(k-5) = 0$$

$$\downarrow \quad \downarrow$$

$$k=0 \quad k=5$$

$$k = (0, 5)$$



(5)

\* Sketch

Q) sketch the graph of  $y = -x^2 + 8x - 12$ step 1:  $a = -1$ 

$a = -1$

vertex is  $V = (x, y)$ 

$b = 8$

$c = -12$

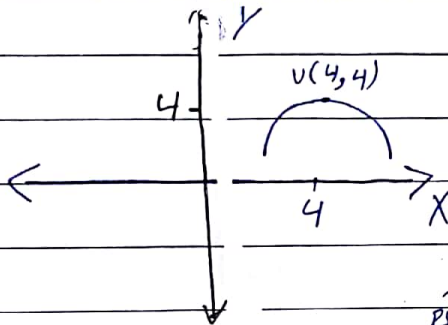
$$\Rightarrow x = \frac{-b}{2a} = \frac{-8}{2(-1)} = \frac{8}{2} = 4$$

$$\Rightarrow y = -(4)^2 + 8(4) - 12 \Rightarrow y = 4$$

 $\Rightarrow$  vertex of the Parabola is  $V = (4, 4)$ 

$a < 0$   
 لأن منقعر للأسفل  
 $a < 0 \rightarrow \text{Maximum}$

step 3:-

 $\Rightarrow$  The graph here is concave down $\Rightarrow$  Maximum value of the Parabola is  $f(4) = 4$ Q) sketch the graph of  $y = f(x) = x^2 - 2x + 3$ 

step 1:

vertex  $V(x, y)$ 

$a = 1$

$$x = \frac{-b}{2a} = \frac{2}{2} = 1$$

$b = -2$

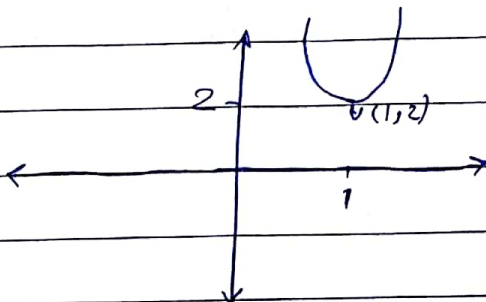
$c = 3$

$$y = (1)^2 - 2(1) + 3 = 2$$

$$\Rightarrow V = (1, 2)$$

 $\Rightarrow$  Minimum value of  $f(x)$  is  $f(1) = 2$ 

$a > 0$  منقعر للأسفل  
 $a > 0 \rightarrow \text{Minimum}$



منقعر للأسفل أو للأسفل  
 $x^2$  منقعر للأسفل

(6)

Q) Find the maximum of the function  $f(x) = -2x^2 + 20x$

sol

$$x = \frac{-b}{2a} = \frac{-20}{2(-2)} = \frac{-20}{-4} = 5$$

$a < 0 \rightarrow \text{Maximum}$

$-2 < 0 \rightarrow \dots$

$$\begin{aligned} \text{Maximum Value} &= f(5) = -2(5)^2 + 20(5) \\ &= -2(25) + 100 = 50 \end{aligned}$$

Q)  $(3x^2 - x) = 0$

sol

$$x(3x - 1) = 0$$

$$\boxed{x = 0} \text{ or } 3x - 1 = 0$$

$$3x = 1$$

$$\boxed{x = \frac{1}{3}}$$

Q) Given the demand and supply functions

$$P = -Q_d^2 - 5Q_d + 52$$

$$P = 2Q_s^2 + 10Q_s + 10$$

$$\boxed{\begin{matrix} \text{Demand} \leftarrow Q_d \\ \text{Supply} \leftarrow Q_s \end{matrix}}$$

Calculate the equilibrium Price and Quantity

sol

At equilibrium  $Q_s = Q_d = Q$

$$-Q^2 - 5Q + 52 = 2Q^2 + 10Q + 10$$

$$(3Q^2 + 15Q - 42 = 0) \div 3$$

$$Q^2 + 5Q - 14 = 0$$

$$(Q + 7)(Q - 2) = 0$$

$$\boxed{Q = -7} \text{ or } \boxed{Q = 2}$$

$$\Rightarrow Q = 2$$

$$\Rightarrow P = 2(2)^2 + 10(2) + 10 = 38$$

$$\Rightarrow (Q, P) = (2, 38)$$

Ques of Quantity ||  
Ques of Price

7.

قوانين

1

$$\textcircled{1} \pi = \text{Profit} = TR - TC$$

Average cost (Ac)

fixed cost (fc)

Price (P)

Quantity (Q)

Variable cost (Vc)

$$\textcircled{2} TR = \text{Total revenue} = P \times Q$$

$$\textcircled{3} TC = \text{Total cost} = FC + VC(Q)$$

$$\textcircled{4} TC = AC(Q) \rightarrow AC = \frac{TC}{Q}$$

Q) If the demand function  $P = 100 - 2Q$  . find TR  
and if  $TR = 0$  find values of Q

$$\begin{aligned} TR &= P \times Q \\ &= (100 - 2Q)(Q) \end{aligned}$$

$$TR = 100Q - 2Q^2$$

$$(-2Q^2 + 100Q = 0) \times -1$$

$$2Q^2 - 100Q = 0$$

$$2Q(Q - 50) = 0$$

$$Q = 0 \text{ or } Q = 50$$

لا تأكلوا الحبوب

حافظوا على أنفسهم من أضرار  
النجاح

Ag



Q) Fixed Cost = 4 and Variable Cost Per unit = 1  
demand function  $P = 10 - 2Q$

1) find Profit  $\pi$   
 $\pi = TR - TC$

Profit  
مربح (القانون)

$$TR = P \times Q$$

$$= (10 - 2Q)(Q)$$

$$TR = 10Q - 2Q^2$$

$$TC = FC + VC(Q)$$

$$= 4 + 1(Q)$$

$$TC = 4 + Q$$

$$\pi = TR - TC$$

$$= 10Q - 2Q^2 - (4 + Q)$$

$$= 10Q - 2Q^2 - 4 - Q$$

$$= 9Q - 2Q^2 - 4$$

$$= -2Q^2 + 9Q - 4$$

2) find The value  $\pi$  if  $Q = 1$

$$\pi = -2Q^2 + 9Q - 4$$

$$\pi = -2(1)^2 + 9(1) - 4 = 3$$

3) find the value of  $Q$  if  $\pi = 0$

$$\pi = -2Q^2 + 9Q - 4$$

$$(0 = -2Q^2 + 9Q - 4) \times -1$$

$$2Q^2 - 9Q + 4 = 0$$

$$(2Q - 1)(Q - 4) = 0$$

$$2Q - 1 = 0 \text{ or } Q - 4 = 0$$

$$2Q = 1$$

$$Q = 4$$

$$Q = \frac{1}{2}$$

The break  
even Point

$$\pi = 0 \text{ when}$$

9

4) find the Max  $\pi$

مطلوب #

$$\pi = -2Q^2 + 9Q - 4$$

$$\text{Vertex} = V = (Q, \pi)$$

$$\left( \frac{-b}{2a}, f\left(\frac{-b}{2a}\right) \right)$$

مطلوب

find the value  $Q$  that.....

vertex

$$Q = \frac{-b}{2a} = \frac{-9}{2(-2)} = \frac{-9}{-4} = \frac{9}{4}$$

$$\pi = -2\left(\frac{9}{4}\right)^2 + 9\left(\frac{9}{4}\right) - 4 = 6.125$$

مطلوب  
Max  $\pi$   
Max TR  
Max TC  
Minimum  $\pi$   
Minimum TR  
Minimum TC

5) Find the value  $Q$  that Max

$$Q = \frac{-b}{2a} = \frac{-9}{2(-2)} = \frac{-9}{-4} = \frac{9}{4}$$

Q)  $\pi = -2Q^2 + 9Q - 4$  and  $fc = 4$  and  $VC$  Per unit = 1  
find the demand function (P).

$$\pi = TR - TC$$

$$-2Q^2 + 9Q - 4 = TR - 4 - Q$$

$$TR = -2Q^2 + 10Q$$

$$TR = P * Q$$

$$\frac{-2Q^2 + 10Q}{Q} = \frac{P * Q}{Q}$$

$$-2Q + 10 = P$$

$$P = -2Q + 10$$

(10)

Q) If  $fc = 25$  and  $VC = 2$  and demand function is

$$P = 20 - Q$$

1) find the Profit function

2) If the Profit = 31, find the value of  $Q$

3) find the Max Profit

$$\text{1) } \pi = TR - TC$$

$$TR = P \times Q$$

$$= (20 - Q)(Q)$$

$$= 20Q - Q^2$$

$$\text{and } TC = fc + VC(Q)$$

$$= 25 + 2Q$$

$$\pi = 20Q - Q^2 - (25 + 2Q)$$

$$= 20Q - Q^2 - 25 - 2Q$$

$$= -Q^2 + 18Q - 25$$

$$2) \pi = -Q^2 + 18Q - 25$$

$$31 = -Q^2 + 18Q - 25$$

$$(-Q^2 + 18Q - 56 = 0) \times -1$$

$$Q^2 - 18Q + 56 = 0$$

$$(Q - 14)(Q - 4) = 0$$

$$Q - 14 = 0 \text{ or } Q - 4 = 0$$

$$\boxed{Q = 14}$$

$$\boxed{Q = 4}$$

$$3) \pi = -Q^2 + 18Q - 25$$

$$V = (Q, \pi)$$

$$Q = \frac{-b}{2a} = \frac{-18}{2(-1)} = \frac{-18}{-2} = 9$$

$$a = -1$$

$$b = 18$$

$$c = -25$$

$$\Rightarrow \pi = -(9)^2 + 18(9) - 25$$

$$= 56$$

56 is the maximum



11

Q) demand function  $P=100-2Q$  find the Max. TR

جواب  $TR = P \times Q$

$$TR = (100 - 2Q)(Q)$$

$$TR = 100Q - 2Q^2$$

$$TR = -2Q^2 + 100Q$$

$$Q = \frac{-b}{2a} = \frac{-100}{-4} = 25$$

$$a = -2$$

$$b = 100$$

$$c = 0$$

سوال #

Max TR |

$$Q = 25$$

$$TR = -2Q^2 + 100Q$$

$$= -2(25)^2 + 100(25)$$

$$= 1250$$

کل شے دے بیچ جلا  
کے ما پر یہ ات زیادہ جلا  
کے اسار افلا

Q)  $\sqrt{\frac{3x}{x-6}} = 3$  is  $x =$

جواب  $\sqrt{\frac{3x}{x-6}} = 3$  is  $x =$

$$\left( \sqrt{\frac{3x}{x-6}} = 3 \right)^2$$

$$\frac{3x}{x-6} = 9 \Rightarrow 9x - 54 = 3x$$

$$6x = 54$$

$$x = 9$$

Q) If  $x^2 - 6x - k = 0$  has exactly one solution then the value  $k$ .

جواب  $\Delta = D = b^2 - 4ac$

$$0 = (-6)^2 - 4(1)(-k)$$

$$0 = 36 + 4k$$

$$4k = -36$$

$$k = -9$$

# \* Rules قوانين

$$1) X^m * X^n = X^{m+n}$$

$$2) \frac{X^m}{X^n} = X^{m-n}$$

$$3) (X^m)^n = X^{m*n}$$

$$4) X^{-n} = \frac{1}{X^n}$$

$$1) A^0 = 1$$

$$2) a^x * b^x = (a*b)^x$$

$$3) \frac{a^x}{b^x} = \left(\frac{a}{b}\right)^x$$

Q:- ①  $-2^3 = -2 * -2 * -2 = -8$

②  $2^3 = 2 * 2 * 2 = 8$

③  $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

4)  $8^{\frac{2}{3}} = (2^3)^{\frac{2}{3}} = 2^{3 * \frac{2}{3}} = 2^2 = 4$

5)  $(16)^{\frac{1}{2}} = (4^2)^{\frac{1}{2}} = 4^{2 * \frac{1}{2}} = 4^1 = 4$

Q:- evaluate

1)  $(32)^{\frac{2}{5}} = (2^5)^{\frac{2}{5}} = 2^{5 * \frac{2}{5}} = 2^2 = 4$

2)  $\left(3 + \frac{2}{4}\right)^{-1} = \left(\frac{14}{4}\right)^{-1} = \frac{4}{14}$

Q:- Solve: 1)  $2^{3x} = 4$

$$2^{3x} = 2^2$$

$$3x = 2$$

$$x = \frac{2}{3}$$

(13)

$$② \quad 8^x = 2\left(\frac{1}{2}\right)^x$$

$$\frac{3x}{2} = 2\left(\frac{1}{2}\right)^x$$

$$\frac{3x}{2} = 2(2^{-1})^x$$

$$\frac{3x}{2} = 2(2)^{-x}$$

$$\frac{3x}{2} = 2^{1-x}$$

$$3x = 1 - x$$

$$4x = 1$$

$$x = \frac{1}{4}$$

$$(2)^2 = 4$$

$$(2)^4 = 16$$

$$2^3 = 8$$

$$(-2)^3 = -8$$

$$(-2)^5 = -32$$

$$\frac{-3}{2} = \frac{1}{8}$$

$$\frac{x}{3} + \frac{x}{3} = 2\left(\frac{3}{3}\right)$$

لا تستسلم  
فكم زفت الأوان  
الآن تبدأ من الآن

\* Logarithm: اللوغاريتم

$$y = \log_a x, \quad x > 0$$

$$a > 1$$

$$a \neq 1$$

# Rules قوانين

$$1) \log_a \frac{x}{y} = \log_a x - \log_a y$$

$$2) \log_a (xy) = \log_a x + \log_a y$$

$$3) \log_a x^n = n(\log_a x)$$

$$4) \log_a x = \log_{10} x$$

$$5) \log_a a = 1$$

$$6) y = \log_a x \Rightarrow x = a^y$$

$$(*) \sqrt[n]{x^m} = x^{\frac{m}{n}}$$

$$(*) \sqrt{x} = x^{\frac{1}{2}}$$

$$(*) \log_a 0 = \text{undefined}$$

$$(*) \log_a -2 = \text{undefined}$$



Q) Solve  $\log_3(3x+1) = 2$

حل  
 $y = \log_x X \Rightarrow X = a^y$

$$3x+1 = 3^2$$

$$3x+1 = 9$$

$$3x = 8$$

$$x = \frac{8}{3}$$

Q) Find the value of

D  $\log_{\frac{1}{4}} 16$

هذا سؤال: نريد نضع (x) إلى ما دة قيمة

حل  $\log_{\frac{1}{4}} 16 = x$

$$\left(\frac{1}{4}\right)^x = 16$$

$$(4^{-1})^x = 4^2$$

$$4^{-x} = 4^2$$

$$\boxed{x = -2}$$

2)  $\log_{\frac{1}{64}} 16$

حل  $\log_{\frac{1}{64}} 16 = x$

$$\left(\frac{1}{64}\right)^x = 16 \Rightarrow \left(4^{-3}\right)^x = 4^2$$

$$4^{-3x} = 4^2$$

$$-3x = 2$$

$$\boxed{x = -\frac{2}{3}}$$

Q) evaluate:-

D  $8^{\frac{4}{3}} = \sqrt[3]{8^4} = \sqrt[3]{8 \times 8 \times 8 \times 8} = \sqrt[3]{4096} = 16$

# Rules: قوانين

D  $\log_e X = \ln X$

4)  $\ln \frac{x}{y} = \ln x - \ln y$

2)  $\ln x^n = n \ln x$

5)  $\ln 1 = 0$

6)  $\ln e = 1$

3)  $\ln(xy) = \ln x + \ln y$

7)  $e^{\ln} = 1$

8)  $\ln e^x = x \ln e = x$

9)  $e^{\ln x} = x$

(15)

Q)  $\log_3 (X-1) + \log_3 (X+1) = 2$  find the value of  $X$ .

Sol  $\log_3 (XY) = \log_3 X + \log_3 Y$

$$\log_3 (X-1)(X+1) = 2$$

$$\log_3 (X^2 - 1) = 2 \Rightarrow X^2 - 1 = 9$$

$$\sqrt{X^2} = \sqrt{10}$$

$$X = \sqrt{10}$$

$$\log_3 X = Y \rightarrow X = 3^Y$$

2)  $\log_{\frac{1}{3}} 27 = ??$

Sol  $\log_{\frac{1}{3}} 27 = X$

$$\left(\frac{1}{3}\right)^X = 27$$

$$\left(\frac{1}{3}\right)^X = 3^3$$

$$3^{-X} = 3^3$$

$$-X = 3$$

$$X = -3$$

ان الفروپ لا ٻڌو ٿو  
ننڍو ٿو ڇڏي

Q) Solve :- 1)  $\log_{10} 100 = \log_{10} 10^2 = 2 \log_{10} 10 = 2$

2)  $\log_{10} \frac{1}{1000} = \log_{10} (1000)^{-1} = \log_{10} (10)^{-3} = -3 \log_{10} 10 = -3$

3)  $\log_{10} 1 = 0$

4)  $\log_{10} 10 = 1$

Q) find value of X

$$1) \log_X (5X-4) = 2$$

لا بد ان يكون داخل اللوغاريتم  
أكبر من 0

$$5X-4 = X^2$$

$$X^2 - 5X + 4 = 0$$

$$(X-4)(X-1) = 0$$

$$X=4 \quad X=1$$

لا بد ان يكون

عدد أكبر من 0  
وهو 4

$$X=4$$

$$\log_X (5X-4) = 2$$

$$\log_4 (16) = 2$$

$$4^2 = 16$$

$$16 = 16$$

$$X=4$$

$$X=1$$

$$\log_X (5X-4) = 2$$

$$\log_1 1 = 2$$

لا تساوي

$$\log_X X = 1$$

$$X=1$$

2)

$$\frac{X}{3} = \frac{2}{2}$$

لا بد ان يكون

$$\ln \frac{X}{3} = \ln \left( \frac{2}{2} \right)$$

$$\ln \frac{X}{3} = \ln 2 - \ln 2$$

$$\ln \frac{X}{3} - \ln 2 = -\ln 2$$

$$X \ln 3 - X \ln 2 = -\ln 2$$

$$X (\ln 3 - \ln 2) = -\ln 2$$

$$X = \frac{-\ln 2}{\ln 3 - \ln 2} = \frac{-\ln 2}{\ln \frac{3}{2}} = \frac{\ln 2}{\ln \frac{2}{3}}$$



(17)

$$\frac{x}{3} = \frac{2^x}{2}$$

$$\log_y x = \frac{\ln x}{\ln y} \rightarrow \text{قاعدة}$$

$$\frac{x}{3} = 2^x$$

$$\ln 2(3)^x = \ln 2^x$$

$$\ln 2 + \ln 3^x = \ln 2^x$$

$$\ln 2 + x \ln 3 = x \ln 2$$

$$\ln 2 = x \ln 2 - x \ln 3$$

$$\ln 2 = x (\ln 2 - \ln 3)$$

$$x = \frac{\ln 2}{\ln 2 - \ln 3} = \log_{\frac{2}{3}} 2$$

لماذا الخوف  
والشمس لا تنظم من ناحية  
الارض وتضيق من ناحية  
اخرى

Q) Simplify تبسيط

$$3 \log_a X - 2 \log_a Y$$

$$\log_a X^3 - \log_a Y^2 \Rightarrow \log_a \frac{X^3}{Y^2}$$

$$2) \log_a X + \log_a Y - \log_a z$$

$$\log_a XY - \log_a z$$

$$\log_a \frac{XY}{z}$$

$$3) \frac{X^3 Y^5}{X^7 Y^2} = \frac{Y^3}{X^4}$$

## # Rules

$$1) e^x \times e^y = e^{x+y}$$

$$2) \frac{e^x}{e^y} = e^{x-y}$$

$$3) e^{\ln x} = x$$

$$4) \ln e^x = x \ln e = x$$

Q) Solve  $2(7)^{-x} = 3^x$

$$2 = \frac{3^x}{7^x}$$

$$2 = (3^x)(7^x)$$

$$2 = (21)^x$$

$$\log 2 = \log 21^x$$

$$\log 2 = x \log 21$$

$$x = \frac{\log 2}{\log 21}$$

$$2) 2^x = 5$$

$$\log 2^x = \log 5$$

$$x = \frac{\log 5}{\log 2}$$

$$2^x = 5$$

$$\ln 2^x = \ln 5$$

$$x \ln 2 = \ln 5$$

$$x = \frac{\ln 5}{\ln 2}$$

(19)

③  $\log(3X+1) = \log(X-2) + 1$

Sol

$$\log(3X+1) - \log(X-2) = 1$$

$$\log\left(\frac{3X+1}{X-2}\right) = 1$$

$$\frac{3X+1}{X-2} = \frac{10}{1}$$

$$3X+1 = 10X-20$$

$$-7X = -21$$

$$X = 3$$

Q) Solve  $e^X = 7$

Sol

$$\ln e^X = \ln 7$$

$$X \ln e = \ln 7$$

$$X = \ln 7$$

Q) write the following in one expression :-

1)  $\log 5 - \log 7 + 2 \log 3$

Sol

$$\log \frac{5}{7} + \log 9$$

$$\log\left(\frac{5}{7} \times 9\right) = \log\left(\frac{45}{7}\right)$$

Q) Solve  $e^{2X} - e^X - 6 = 0$

Sol

$$(e^X - 3)(e^X + 2) = 0$$

$$e^X = 3 \text{ or } e^X = -2$$

$$\ln e^X = \ln 3$$

$$X \ln e = \ln 3$$

$$X = \ln 3$$

$$-2 < 0$$



(20)

$$① y = \log X$$

$$② y = \ln X = \log_e X$$

$$3) y = a^x \quad (Y = 3^x)$$

$$4) y = e^x$$

$$Q) \text{ Solve } 2^{x+1} + 2^{x-1} = 20$$

$$\underline{\text{Sol}} \quad (2^x)(2)^1 + (2^x)(2)^{-1} = 20$$

$$(2^x)(2^1 + 2^{-1}) = 20$$

$$(2^x)(2 + \frac{1}{2}) = 20$$

$$((2^x)(\frac{5}{2}) = 20) \times \frac{2}{5}$$

$$\frac{2^x}{2} = 8$$

$$\frac{2^x}{2} = 2^3$$

$$\boxed{X = 3}$$

$$Q) \text{ Solve } 3^{2X+1} \leq 7$$

$$\underline{\text{Sol}} \quad \log 3^{2X+1} \leq \log 7$$

$$(2X+1)\log 3 \leq \log 7$$

$$2X \log 3 + \log 3 \leq \log 7$$

$$2X \log 3 \leq \log 7 - \log 3$$

$$2X \log 3 \leq \log \frac{7}{3}$$

$$(2X \leq \frac{\log \frac{7}{3}}{\log 3}) \times \frac{1}{2}$$

$$X \leq \frac{\log \frac{7}{3}}{2 \log 3} \Rightarrow X \leq \frac{\log \frac{7}{3}}{\log 9}$$

$$\log_b a = \frac{\log a}{\log b}$$

(21)

Q) If  $Y = \ln 5 + \ln(X-3)$  write  $X$  in terms of  $Y$

Sol  $Y - \ln 5 = \ln(X-3)$

$$\frac{(Y - \ln 5)}{e} = \ln(X-3)$$

$$\frac{(Y - \ln 5)}{e} = X - 3$$

$$\frac{(Y - \ln 5)}{e} + 3 = X$$

Q) Solve  $3\left(2^X\right) = 10^X$

Sol  $\log 3\left(2^X\right) = \log 10^X$

$$\log 3 + \log 2^X = \log 10^X$$

$$\log 3 + X \log 2 = X \log 10$$

$$X \log 2 - X \log 10 = -\log 3$$

$$X (\log 2 - \log 10) = -\log 3$$

$$X (\log 2 - 1) = -\log 3$$

$$X = \frac{-\log 3}{\log 2 - 1}$$

Q) Solve  $\ln(2x) - \ln(x-1) = 0$

الحل

$$\ln \frac{2x}{x-1} = 0$$

$$\frac{\ln \frac{2x}{x-1}}{e} = e^0$$

$$\frac{2x}{x-1} = 1$$

$$2x = x - 1$$

$$\boxed{x = -1}$$
 ← <sup>مقبولة</sup> <sup>كـ</sup>

2)  $\ln x + \ln(x-1) = 0$

$$\ln(x)(x-1) = 0$$

$$\ln(x^2 - x) = 0$$

$$\frac{\ln(x^2 - x)}{e} = e^0$$

$$x^2 - x = 1$$

$$a=1, b=-1, c=-1$$

$$x^2 - x - 1 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$D = b^2 - 4ac$$
$$= 1 - 4(1)(-1)$$
$$= 5$$

$$\boxed{x = \frac{1 + \sqrt{5}}{2}} > 0$$

↓  
مقبول

$\ln 0$  غير مقبولة  
 $\ln$  سالب

$$\boxed{x = \frac{1 - \sqrt{5}}{2}} < 0$$

↓  
مرفوض (أبذق)



(23)

$$Q) \frac{4(2)^x}{(\sqrt{2})^6} = 16, \text{ then } X$$

$$\text{d.s.} \frac{2^2(2)^{3x}}{(2^{\frac{1}{2}})^6} = 16$$

$$\frac{(2^2)(2)^{3x}}{2^3} = 2^4$$

$$\frac{2+3x-3}{2} = 2$$

$$\frac{3x-1}{2} = 2$$

$$3x-1=4$$

$$3x=5$$

$$\boxed{x = \frac{5}{3}}$$

$$Q) \ln(12-x^2) = \ln 1 + \ln x$$

$$\boxed{\ln 1 = 0}$$

$$\text{d.s.} \ln(12-x^2) = \ln x$$

$$\frac{\ln(12-x^2)}{e} = \frac{\ln x}{e}$$

$$12-x^2 = x$$

$$x^2 + x - 12 = 0$$

$$(x-3)(x+4)$$

$$\text{جواب} \rightarrow \boxed{x=3} > 0$$

$$\text{جواب} \rightarrow \boxed{x=-4} < 0$$

Q/  $\log_y X = 15$  and  $\ln X = 5$  then  $\ln y$

$$y^{15} = X$$

$$\ln X = 5$$

$$X = e^5 \Rightarrow y^{15} = e^5$$

$$y^{15} = e^5$$

$$\ln y^{15} = \ln e^5$$

$$15 \ln y = 5 \ln e$$

$$15 \ln y = 5$$

$$\ln y = \frac{5}{15} = \frac{1}{3}$$

Q)  $\log_2 (9^{x-1} - 27) = 2$

$$9^{x-1} - 27 = 4$$

$$9^{x-1} = 27$$

$$\frac{2(x-1)}{3} = \frac{3}{3}$$

$$\frac{2x-2}{3} = \frac{3}{3}$$

$$2x-2=3$$

$$2x=5$$

$$x = \frac{5}{2}$$

(25)

Q)  $\ln X + \ln 2X = 4$  then X

Sol

$$\ln(X \times 2X) = 4$$

$$\ln(2X^2) = 4$$

$$e^{\ln(2X^2)} = e^4$$

$$2X^2 = e^4$$

$$X^2 = \frac{e^4}{2}$$

$$X = \sqrt{\frac{e^4}{2}} = \frac{e^2}{\sqrt{2}}$$

Q)  $2e^X - e^{2X} = 0$  then X

Sol

$$\frac{2e^X}{e^X} = \frac{e^{2X}}{e^X}$$

$$2 = e^{2X-X}$$

$$2 = e^X$$

$$\ln 2 = \ln e^X$$

$$X = \ln 2$$

Q) Find the value X

D  $\frac{3 \ln(2X)}{e} = ??$

Sol  $\frac{\ln(2X)^3}{e} = ?$

$$(2X)^3 = 8X^3$$



Q)  $10^X \times 10^1 = \frac{3}{2}$

Ans  $(10^X \times 10^1 = \frac{3}{2}) \times \frac{1}{10}$

$$10^X = \frac{3}{2} \times \frac{1}{10}$$

$$10^X = \frac{3}{20}$$

$$\log 10^X = \log \frac{3}{20}$$

$$\log 10 = 1$$

$$X \log 10 = \log \frac{3}{20}$$

$$X = \log \frac{3}{20}$$

Q)  $\log_{\frac{8}{32}} \frac{1}{32}$

Ans  $\log_{\frac{8}{32}} \frac{1}{32} = X$

$$\frac{8}{32}^X = \frac{1}{32}$$

$$\frac{1}{2} = 2^{-5}$$

$$3X = -5$$

$$X = \frac{-5}{3}$$

Q) If  $2 \log X = \log (21 + 4X)$ , then  $X =$

Q) If  $\log_2 (2X-2) = 3$  then  $X$

Q) If the average Cost function of a good  $AC = \frac{15}{Q} + 6Q$   
then the total cost function when  $Q=2$  is  $TC$

Ans

$$AC = \frac{TC}{Q} \Rightarrow TC = AC(Q)$$

$$TC = AC(Q)$$

$$\left(\frac{15}{Q} + 6Q\right)(Q) = 15 + 6Q^2$$

$$TC|_{Q=2} \Rightarrow 15 + 6(2)^2 = 39$$

Q) If  $5^x = 3^{2x+1}$  then  $x$

لن نضرب

الكل في

$$Q) \log_6(3) + \log_6(24) - \log_6\left(\frac{1}{3}\right) =$$

Ans

$$\log_6 \frac{(3)(24)}{\frac{1}{3}}$$

$$\log_6 (3)(24)(3)$$

$$\log_6 216 = x$$

$$6^x = 216$$

$$6^x = 6^3$$

$$\boxed{x = 3}$$

Q) If  $f(x) = x^2 + 3x + 2$  find  $f(3+a)$

Ans

$$f(3+a) = (3+a)^2 + 3(3+a) + 2$$

$$a^2 + 9a + 20$$

(28)

Q)  $\ln X + \ln(2X) = \ln 8$  then X.

Ans

$$\ln(X)(2X) = \ln 8$$

$$\ln(2X^2) = \ln 8$$

$$\frac{\ln(2X^2)}{e} = \frac{\ln 8}{e}$$

$$2X^2 = 8$$

$$X^2 = 4$$

$$X = +2 \rightarrow \text{correct}$$

$$X = -2$$

correct

Q) 15)  $\frac{1}{2} \log 9 + \frac{1}{3} \log 1000 + \log 3$  is

Ans

$$\log \sqrt{9} + \log \sqrt[3]{1000} + \log 3$$

$$\log 3 + \log 10 + \log 3$$

$$\log 9 + 1$$

Q) If  $9^X = 3(3)^{-X}$  find then the value X

Ans

$$\frac{2X}{3} = \left(\frac{1}{3}\right) \left(\frac{-X}{3}\right)$$

$$\frac{2X}{3} = \frac{1-X}{3}$$

$$2X = 1-X$$

$$X = \frac{1}{3}$$



(29)

Q)  $\log_{10}(X-Y) = 3$  ,  $\log_{10}(X+Y) = 3$  Find the value X

Ans

$$10^3 = X - Y$$

$$10^3 = X + Y$$

$$1000 = X - Y$$

$$1000 = X + Y$$

$$X - Y = 1000$$

$$X + Y = 1000$$

$$2X = 2000$$

$$X = 1000$$

Q) The value of  $\ln \left( \frac{e^4 e^{\frac{2}{3}}}{\sqrt{e}} \right)$

Ans  $\ln \frac{e^{\frac{12}{3} + \frac{2}{3}}}{e^{\frac{1}{2}}}$

$$\ln \frac{e^{\frac{14}{3}}}{e^{\frac{1}{2}}}$$

$$\ln e^{\frac{14}{3} - \frac{1}{2}}$$

$$\ln e^{\frac{28}{6} - \frac{3}{6}}$$

$$\ln e^{\frac{25}{6}}$$

$$\frac{25}{6} \ln e \Rightarrow \frac{25}{6}$$

30

$$Q) \log(3X+1) = \log(X-2) + 1$$

sol

$$\log(3X+1) - \log(X-2) = 1$$

$$\log_{10} \frac{3X+1}{X-2} = 1$$

$$10^1 = \frac{3X+1}{X-2}$$

$$10X - 20 = 3X + 1$$

$$\boxed{X=3}$$

$$Q) \log_b 2 = k, \log_b 3 = s, \log_b 10 = r$$

$$\log_b 180 = ?? \Rightarrow \log_b (3 \times 3 \times 2 \times 10)$$

$$= \log_b 3 + \log_b 3 + \log_b 2 + \log_b 10$$

$$= s + s + k + r$$

$$2s + k + r$$

Q1) If  $2^{3X} = 2\left(\frac{1}{2}\right)^X$ , then  $X =$

Q2) If  $2e^X - e^{3X} = 0$ , then

$$e^X(2 - e^{2X}) = 0 \Rightarrow e^{2X} = 2$$

$$\ln e^{2X} = \ln 2$$

$$2X = \ln 2$$

$$X = \frac{\ln 2}{2}$$

Q3) If  $\log_{10}(3X+2) = 2$  then  $X =$

Q4) If the equation  $X^2 + 4X = m$  has only one solution then  $m =$

Q5) The maximum Profit  $\pi = -Q^2 + 12Q - 10$ , is  $\pi =$

Q6) Given the supply and demand functions

$$P = Q_s^2 + 14Q_s + 22 \text{ and } P = -Q_d^2 - 10Q_d + 150$$

Find the equilibrium Price and Quantity

Q7) The value of  $k$  such that the equation

$$X^2 - 5X + 2k = X - k \text{ has only one solution}$$

Q8)  $\log_X(3X+10) = 2$ , then  $X =$

A) -2, 5    B) -2    C) 5    D) 2    E) None

Q9) The solution of  $(81)^X = (3)^{2X-1}$  is

Q10) If  $\frac{(X+1)}{3} = 5$  then  $X =$

A)  $\log_{\frac{3}{5}} \frac{5}{3}$     B)  $\log_{\frac{5}{3}} \frac{5}{3}$     C)  $\log_{\frac{5}{3}} \frac{5}{3}$     D)  $\log_{\frac{3}{5}} \frac{3}{5}$     E) None



Q11) If the Profit function is  $\pi = -3Q^2 + 42Q - 5$   
and  $TC = 5 + 4Q$  then the demand function  $P =$

Q12) If the Profit function is  $\pi = -3Q^2 + 6Q + 5$   
then the value of  $Q$  that Maximize the Profit is

Q13)  $8^x = 4\left(\frac{1}{2}\right)^x$ , then  $x =$

Q14)  $2e^x - e^{2x} = 0$ , then  $x =$

Q15) If  $\log_x(2x+3) = 2$

Q16) If the equation  $x^2 - 2x = c$  has only one solution, then  $c =$

Q17) The Max Profit  $\pi = -Q^2 + 18Q - 25$ , is  $\pi =$

Q18) If the equation  $x^2 + 7x + 3c = -3x - 2c$   
has exactly one solution, then the value of  $c =$

Q19) If  $\log_3(2^{x+1} + 49) = 4$ , then  $x =$

Q20) If  $(12 - x^2) = \ln 1 + \ln x$ , then  $x =$

Q21) If  $\ln x + \ln 2x = 4$ , then  $x =$

Q22) If  $\log_3(2x-3) = 2$ , then  $x =$

Q23) If  $X^2 - 3X + 10 = 2X + 4$  then  $X =$

Q24) Given the demand and supply function

$P = -Q_d^2 - 5Q_d + 52$ ,  $P = 2Q_s^2 + 10Q_s + 10$   
then the equilibrium Price is  $P =$

Q25) If the Profit function  $\pi = -2Q^2 + 9Q - 4 = 0$   
then the values of  $Q$  is (are)  $Q =$

Q26)  $(81)^{\frac{-3}{4}}$

Q27) The value of  $\log_{\frac{1}{3}} 27$

Q28) If  $(X+2)^2 + (2X-1)^2 = X(4X+6)$ , then  $X =$

Q29)  $3 + \log_{10} X = \log_{10} 100$ , then  $X =$

Q30) given supply and demand functions

$P = -Q + 4$ ,  $P = 2Q^2 + Q - 11$  then the Point equilibrium

If the government imposes a fixed tax of 3 T/d each good is =

Q31) Function Profit  $\pi = -5Q^2 + 20Q$  and  $TC = 5 + 2Q$   
find TR

Q32)  $\ln X - \ln 3 = 2$  find  $X$

Sol

$$\ln \frac{X}{3} = 2$$

$$\frac{\ln X}{e} = \frac{2}{e}$$

$$\frac{X}{3} = e^2$$

$$X = 3e^2$$

Q33)  $\frac{(x+1)}{2} = 7$

$\log_2 \frac{(x+1)}{2} = \log 7$

$(x+1) \log 2 = \log 7$

$x \log 2 + \log 2 = \log 7$

$x \log 2 = \log 7 - \log 2$

$x = \frac{\log \frac{7}{2}}{\log 2} \Rightarrow x = \log_2 \frac{7}{2}$

Q34) If the equation  $x^2 + 7x + 4k = -3x - k$  has one solution then the value of k

Q35)  $\pi = -4Q^2 + 40Q - 6$  find that maximize

Q36)  $\frac{(x-1)}{3} = 5$

Q37) If the Profit function  $\pi = -4Q^2 + 2Q - 3$

and  $TC = 3 + 2Q$  find demand function P

Q38)  $\log(3x+1) = \log(x+2) + 1$  find X

Q39)  $\frac{(x+1)}{2} \leq 3$  find X

Q40)  $\log X = \log 1000 - 2$  find X

Q41) If  $10^{\frac{3x}{y}} = \frac{4}{5}$  find Y when X=1

Q42)  $\ln X - 4 =$  find X



Q43)  $\ln X + \ln 3X = 4$  find  $X$

Q44)  $9^X = 3^X \left(\frac{1}{3}\right)^X$  find  $X$

Q45) find  $\ln\left(\frac{e^4}{\sqrt{e}}\right)$

Q46) Solve  $\log_3\left(3^{X+1} - 18\right) = 2$

Q47) Solve  $\log(X^2 + 9X) = 1$

Q48)  $X + \frac{1}{X} = 7$  then  $X^2 + \frac{1}{X^2} =$

Q49)  $\pi = -6Q^2 + 24Q + 5$  find  $\text{Max } \pi$

بالتوفيق  
وهيك بيتكون انتفت المادة  
ان شاء الله اكون شملت كل جزئية بالمادة  
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